ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA | DEPARTMENT
OF MATHEMATICS

Activity plan and research project of "*Chow Ring of Toric Arrangements*"

These postdoc positions are funded by the homonymous project FIS 2 CRoTA "*Chow Ring of Toric Arrangements*" FIS-2023-00808, CUP: J53C25001900001.

Research project

This project addresses a fundamental question on the relation between combinatorics and geometry. Mathematicians can prove theorems in combinatorics using algebraic geometry. Vice versa, we can do algebraic geometry also in absence of the algebraic variety by using combinatorics. These leitmotifs ware recently discovered by the field medallist June Huh in a **linear setting**.

The question is: does Huh's philosophy depend on the linear setting? Does it apply in much broader generality? The project will answer this question by studying the **toric case**. This case is the middle point between the linear setting and the generic case.

The project is multidisciplinary sitting between combinatorics, geometry, algebra, and topology. PI's knowledge in the field of **arrangements** and **matroids** is an almost unique and necessary mix to carry out this research.

The aim is to show that almost all results about matroids – obtained in the last five years using algebraic geometry – can be extended to arithmetic matroids. The research will also increase the knowledge of toric arrangements and make them a new toy example and a new local model for all geometric theories. Concretely, the research group will **invent a machinery** that for almost any poset produces an algebra, theorems for that algebra, and inequalities between the numerical invariants of the given poset.

Methods used in this research will be various, including sheaves on posets, Chern theory, toric and tropical geometry. The last part of the project consists in applications of the machinery to combinatorics and algebraic geometry; in particular to polytope theory, resolution of singularities and moduli spaces.

The main idea is to define a class of posets that behaves like the poset of flats of a toric arrangement; from that poset construct a **Chow ring of the poset**, prove the Hodge package, the relative Lefschetz decomposition, and describe the NEF cone. Then, the research group will define and study important classes in the Chow ring of the poset such as CSM classes and Chern classes of a tautological bundle.

Finally, the **intersection numbers** of these classes will be related to the combinatorics of the poset. The aforementioned theorems, inspired by the algebraic geometry, will provide new inequalities for such numerical invariants.

Activity plan

The candidate expertise should match one of the following profile and his/her activities will depend on his/her profile.

Profile 1: He/she is an expert in **algebraic combinatorics** with knowledge in combinatorial structures such as poset and simplicial complexes and related rings (e.g., Stanley Reisner rings).
Activities: The postdoc will apply his/her skills on the poset of flats of a toric arrangement and the associated order complex, with particular focus on blowup of posets and cohomology ring of the variety.

Profile 2: He/she has knowledge about **wonderful models**. More generally, the postdoc has familiarity with algebraic varieties defined from combinatorial objects (e.g., Grassmannian varieties) and with their invariants (e.g., cohomology and Chow ring, intersection theory).
Activities: The postdoc will apply his/her skills on the toric wonderful models and their ample cones. Finally, he/she studies the CSM classes of toric arrangements, thanks to previous knowledge in intersection theory.

Profile 3: He/she is an expert in **tropical geometry** with knowledges about polyhedral complexes, Bergman fan, and Minkowski weights.
Activities: The postdoc will apply his/her skills on the toric wonderful model by giving a tropical description of it and applying the theory of tropical varieties to the model. In particular, he/she would obtain a description of the NEF cone.

Profile 4: He/she is a **toric geometer** with skills in combinatorics (e.g., polytopes and fans) and in algebraic geometry (e.g., vector bundles and Chern classes).
Activities: The postdoc will apply his/her skills in two ways: by computing the CSM of open subsets of toric varieties (e.g., union of orbits and complement of toric arrangements) and constructing a toric parameter space for toric arrangements.

Profile 5: He/she is an **algebraic geometer** with knowledge in sheaves theory, cohomology theories, and decomposition theorems (e.g., the Hard Lefschetz theorem).
Activities: The postdoc will apply his/her skills to understand the relative Lefschetz decomposition for toric wonderful models.

Profile 6: He/she is an **algebraic topologist** with skills in hyperplane arrangements, arrangements of pseudospheres, homological and cohomological invariants, Orlik-Solomon algebras, operads and categories.
Activities: The postdoc will apply his/her skills about pseudo-arrangements to understand the topological and combinatorial requirement for the non-realizable object. In the last year he/she contributes to the problem of resolution of singularities with his/her knowledge on cohomology rings.

Profile 7: He/she is an expert of **matroid theory** with a knowledge of Tutte polynomial, oriented matroids, arithmetic matroids, lattices, and possibly log-concavity properties.

Activities: The postdoc will apply his/her skills to arithmetic matroids, their generalizations, and their numerical invariants with focus on unimodality properties.

Profile 8: He/she is an expert of **discrete combinatorics**. The apport of this member consist in the expertise on polytopes, poset, enumerative aspects of polytopes (e.g., h-vector and f-vector), and numerical sequences.
Activities: The postdoc will apply his/her skills to fans, piecewise-linear functions on their supports and poset blowups. Later, he/she focused on numerical properties of arithmetic Tutte polynomial and Ehrhart polynomial of zonotopes.

Profile 9: He/she is an **algebraist**, possibly interested in the infinite dimensional algebras, representation theory, commutative algebra, and homological algebra.
Activities: The postdoc will apply his/her skills to deal with the infinite model and its properties. The expertise in representation theory will help the research in the relative Hard Lefschetz by studying the symmetries of the problem.

Profile 10: He/she is an **enumerative algebraic geometer,** or a geometer interested in moduli spaces. The postdoc has familiarity with the algebraic geometry language (sheaves, schemes, stacks…) and interest in enumerative properties (e.g., intersection numbers, configuration of special varieties, finite fields method…).
Activities: The postdoc will apply his/her skills to extend the intersection properties of toric wonderful model to the non-realizable case. Finally, he/she will be essential for the application to moduli spaces.